

A discussion of deuteron transverse charge densities

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The deuteron transverse charge density $\rho_C(b)$ is the two-dimensional Fourier transform of its charge form factor in the impact space. We show that different parameterizations of the charge form factors provide different $\rho_C(b)$, in particular at the central value of impact parameter ($b = 0$), although all the parameterizations can well reproduce the form factors in the region of small Q^2 . In addition, we also check the explicit contributions from the different coordinate intervals of the deuteron wave function to its root-mean-square radius.

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I. INTRODUCTION

The deuteron form factors have received much attention and have been extensively discussed for several decades (for some recent reviews, see, e.g. [1–4]). A deuteron, as a spin-1 particle, has three form factors of charge G_C , magnetic G_M , and quadrupole G_Q . It is often regarded as a loosely bound state of the proton and neutron (with binding energy $\epsilon_D \sim 2.22$ MeV), and consequently the study of the deuteron properties can shed light on the structure of the nucleon (in particular of the neutron) as well as of the nuclear effects. The deuteron electromagnetic (EM) properties can be explored by the lepton-deuteron elastic scattering. The matrix element of the electron-deuteron (eD) elastic scattering in the one-photon approximation, as shown in Fig. 1, can be written as

$$\mathcal{M} = \frac{e^2}{Q^2} \bar{u}_e(k') \gamma^\mu u_e(k) \mathcal{J}_\mu^D(P, P'), \quad (1)$$

where k and k' are the four-momenta of initial and final electrons. $\mathcal{J}_\mu^D(P, P')$ is the deuteron EM current, and its general form is

$$\mathcal{J}_\mu^D(P, P') = - \left(G_1(Q^2) \epsilon'^* \cdot \epsilon - \frac{G_3(Q^2)}{2M_D^2} \epsilon \cdot q \epsilon'^* \cdot q \right) (P + P')_\mu - G_2(Q^2) \left(\epsilon_\mu \epsilon'^* \cdot q - \epsilon'_\mu \epsilon \cdot q \right), \quad (2)$$

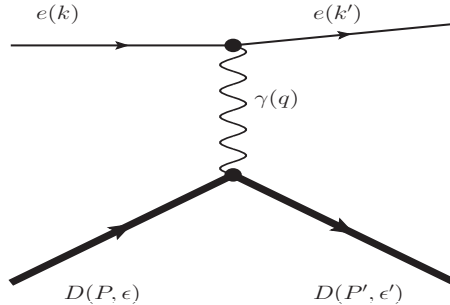


FIG. 1: Electron-deuteron elastic scattering in the one-photon approximation.

where M_D is the deuteron mass, $\epsilon(\epsilon')$ and $P(P')$ are the polarization and the four-momentum of the initial (final) deuteron, and $Q^2 = -q^2$ is the momentum transfer square with $q = P' - P$. The three EM form factors $G_{1,2,3}$ of the deuteron are related to the charge G_C , magnetic G_M , and quadrupole G_Q form factors by

$$G_C = G_1 + \frac{2}{3}\tau G_Q, \quad G_M = G_2, \quad G_Q = G_1 - G_2 + (1 + \tau)G_3, \quad (3)$$

with $\tau = \frac{Q^2}{4M_D^2}$.

Since the deuteron is an isoscalar particle, if one only consider the contribution of the isoscalar vector mesons- ω and ϕ , that the parameterizations of the deuteron form factors can be written as [5]

$$G_i(Q^2) = N_i g_i(Q^2) F_i(Q^2), \quad i = C, Q, M, \quad F_i(Q^2) = 1 - \alpha_i - \beta_i + \alpha_i \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_i \frac{m_\phi^2}{m_\phi^2 + Q^2}, \quad (4)$$

where m_ω (m_ϕ) is the mass of the ω (ϕ)-meson. Note that the Q^2 -dependence of $F_i(Q^2)$ is parameterized in such form that $F_i(0) = 1$, for any values of the free parameters α_i and β_i , which are real numbers. The terms $g_i(Q^2)$ can be written as the functions of the two real parameters of γ_i and δ_i , which are generally different for each form factor

$$g_i(Q^2) = 1 / [1 + \gamma_i Q^2]^{\delta_i}, \quad (5)$$

and N_i is the normalization of the i -th form factor at $Q^2 = 0$:

$$N_C = G_C(0) = 1, \quad N_Q = G_Q(0) = M^2 \mathcal{Q}_D = 25.83, \quad N_M = G_M(0) = \frac{M_D}{M_N} \mu_D = 1.714, \quad (6)$$

where \mathcal{Q}_D , and μ_D are the quadrupole and the magnetic moments of the deuteron, and M_N is the nucleon mass. It should be reiterated that this parametrization scheme has been discussed explicitly by Ref. [5].

There are a lot of studies on the EM properties of the nucleon and deuteron, like their EM form factors, in the literature. Recently, the nucleon transverse charge density attracts great interest, since this two-dimensional density can directly relate to the matrix element of a density operator. It stands for the two-dimensional Fourier transform of the EM form factor and presents for the density (in the infinite momentum frame) located at a transverse separation b (impact parameter) from the center of transverse momentum [6–10]. The transverse densities of the pion and nucleon have been discussed extensively. In particular, the transverse density of pion at central b ($b = 0$) could tell from the different model calculations because theoretical predictions give different behaviors at $b = 0$, even if they all can well reproduce the EM form factors of the pion [11].

On the other hand, the root-mean-square (*rms*) radius of the proton $R = \langle r^2 \rangle^{1/2}$ has also become particular interest recently. This is because the inconsistency between the measurements of the ep elastic scattering and of the hyperfine structure of the muonic hydrogen atom. The proton size is, measured by the ep elastic scattering experiment, $(0.879 \pm 0.005 \pm 0.004)$ fm [12]. However, the *rms* radius can also modify the hydrogen energy levels. With the 2P-1S transition energy and the 1S hyperfine structure of hydrogen, which are well measured with a high degree of accuracy, the *rms* radius of the proton can be extracted and the obtained value is 0.8678 ± 0.0069 fm [13]. Similar to hydrogen, one can also get the information of the proton *rms* radius from muonic hydrogen. The short lifetime and heavy mass of the muons make the muonic hydrogen energy levels being more sensitive to the proton *rms* radius than the hydrogen. The recent extracted *rms* radius of the proton is 0.84184 ± 0.00067 fm, which is extracted from the hyperfine structure of the muonic hydrogen atom [14]. This value is about 5σ smaller than the ones from ep elastic scattering and from hydrogen spectroscopy. After this inconsistency was found, more precise investigation of the *rms* radius of the proton becomes necessary. There are many parameterizations of the proton form factors in the literature, which all fit the data well. Detailed analysis for the different parameterizations shows that the contributions of the wave function in the large distance region is sizable [15]. Therefore, the *rms* radius of the proton is also sensitive to the wave function in the large distance.

In this work, analogous to the pion and the proton targets, the deuteron transverse charge density $\rho_C(b)$ is analyzed with the help of several sets of parameterizations of the deuteron form factors which have been given by Ref. [5]. It should be mentioned that those parameterizations are employed to fit the measured deuteron form factors, and therefore, in the low Q^2 region, one expects that those all match the data well. However, when we make the Fourier transform of the form factors in order to get the densities in the impact parameter space, the densities in the region of small b (particularly in the central $b = 0$) are expected to be dominantly affected by the form factors in the large Q^2 region. Therefore, the impact parameter b -dependence of the transverse densities provides more information for the form factors in the large Q^2 region. In addition, the deuteron wave function of the Paris N-N potential [16] will be employed to re-study the *rms* radius of the deuteron and particularly to see the relation of the *rms* radius of the deuteron to the wave function in the large distance (say $r > 4$ fm) region.

II. PHENOMENOLOGICAL ANALYSIS

The density $\rho_C(b)$ of deuteron in the transverse plane is the two-dimensional Fourier transform of its EM form factors. The Fourier transform of the charge form factor is (see the detailed calculation of the transverse charge density in Refs. [11, 17])

$$\rho_C(b) = \frac{1}{(2\pi)^2} \int d^2Q G_C(Q^2) e^{i\vec{q}\cdot\mathbf{b}}, \quad (7)$$

where $Q^2 = -q^2 = \vec{q}^2 > 0$ in the Drell-Yan frame [11] ($q^+ = 0$), b is the impact parameter in the two-dimensional transverse plane. Using Eqs. (4) and (5) with the parameters given by Ref. [5], we get the Tables (I-IV), where the contributions by the integral from the Q intervals of $(0-\infty)$, $(0-1)$, $(1-2)$, $(2-10)$, and $(10-\infty)$ GeV are separately displayed. Fig. 2 gives the estimated two dimensional transverse charge density $\rho_C(b)$ in the impact b -space. Fig. 2 tells that although all the four parameterizations of the deuteron charge form factors fit the available data in the small Q^2 region well, they provide very different $\rho_C(b)$, particularly in the small center b -value. The scheme I of the parameterizations gives infinity for $\rho_C(b=0)$ (see Table I), while other three give 0.72, 1.42, and 0.90 fm⁻² in the limit of $b=0$, respectively. The divergence of the transverse charge densities at $b=0$ is expected to be related to the parameterizations of $g_i(Q^2) = 1/[1 + \gamma_i Q^2]^{\delta_i}$ (see Eq. (5)). If δ_i is smaller than 1, then, the integral with respect to Q^2 (see Eq. (7)) at $b=0$ limit turns to infinity. Therefore, the $b=0$ behavior of the transverse charge density is dominantly contributed by the charge form factor in the large Q^2 region, and it determines the form factor in the large Q^2 region. Actually, this feature also can be seen from Tables (I-IV).

We see that there are remarkable discrepancies between the different parameterizations at the small value of b , particularly at $b=0$, and the discrepancies become smaller when b increases. When the b becomes smaller, the contributions from large Q^2 region to the integral of Eq. (7) turn to be more important. This is not surprising. At large Q^2 region which corresponds to small b regime, different model parameterizations of $G_C(Q^2)$ are not always the same. Some $\rho_C(b)$, at central b , have definite numbers, and some are divergent. Different parameterizations of the charge form factors provide very different $\rho_C(b)$. We find from Tables (I-IV) that the contribution to $\rho_C(b)$ of $G_C(Q^2)$ in the region $10 \leq Q \leq \infty$ GeV becomes sizeable when b is decreasing. In particular, in the region of very small b -value, say less than 0.05 fm, the contribution turns dominant. Substituting the parameters of different model parameterizations (the parameters in Tables (I-IV)) into Eqs. (4) and (5), we obtain the ratios of the estimated $\rho_C(b)$ in the region of $Q = (1-2)$ GeV to the one in the whole Q region ($Q = (0-\infty)$ GeV)

$$\begin{aligned} \text{Ratio} &= \frac{\int_0^2 \frac{d^2q}{(2\pi)^2} G_C(\vec{q}^2) e^{i\vec{q}\cdot\mathbf{b}}}{\int_0^\infty \frac{d^2q}{(2\pi)^2} G_C(\vec{q}^2) e^{i\vec{q}\cdot\mathbf{b}}} \\ &= \frac{\int_0^2 \frac{qdq}{2\pi} G_C(\vec{q}^2) J_0(qb)}{\int_0^\infty \frac{qdq}{2\pi} G_C(\vec{q}^2) J_0(qb)}, \end{aligned} \quad (8)$$

see Fig. 3. It shows more clearly that when b decreases, the contribution from large Q^2 region increases, while the contribution from small Q^2 region (say $0 < Q < 2$ GeV) decreases. So a well understanding of the form factor in the large Q^2 region is necessary if one wants to know the transverse charge density near the central value of b in the impact space. It also means that the transverse charge density at the central b sheds light on the form factor in the large Q^2 region. Whether the model parametrization is good or not also can be judged by the estimated $\rho_C(0)$, particularly in the small b region. So far, our knowledge of the form factors in the large Q^2 regime is rather limited. More precise experimental data are needed in order to determine the tail of the form factor in the region.

In the region of moderated b (say 0.5 ~ 1 fm), the estimated transverse charge density of $\rho_C(b)$ is expected to be dominated by the form factor in the region of $Q \sim 1$ GeV, where meson cloud effect on the charge form factors of the proton and neutron is expected to be important. Conventionally, we consider the three quark core $|3q\rangle$ in the proton or neutron, the core is always located in the central b and the size of the core is expected to be smaller than 0.5 fm. When the meson cloud, pion meson cloud for example, is considered, the proton has the components of $|(3q)^0\pi^+\rangle$ and $|(3q)^+\pi^0\rangle$ and the neutron has the components of $|(3q)^+\pi^-\rangle$ and $|(3q)^0\pi^0\rangle$. Thus, the long positive tail of the proton transverse charge density comes from the positive pion cloud and on the contrary, the long negative tail of the neutron transverse charge density results from the negative charged pion cloud [18]. According to our estimate, the obtained $\rho_C(b)$ all have a positive long tail. This is due to the positive proton tail.

In addition, one usually estimates the deuteron root-mean-square radius from the deuteron model-dependent wave function

$$r_d = \frac{1}{2} \left\{ \int_0^\infty dr r^2 [u^2(r) + w^2(r)] \right\}^{\frac{1}{2}}. \quad (9)$$

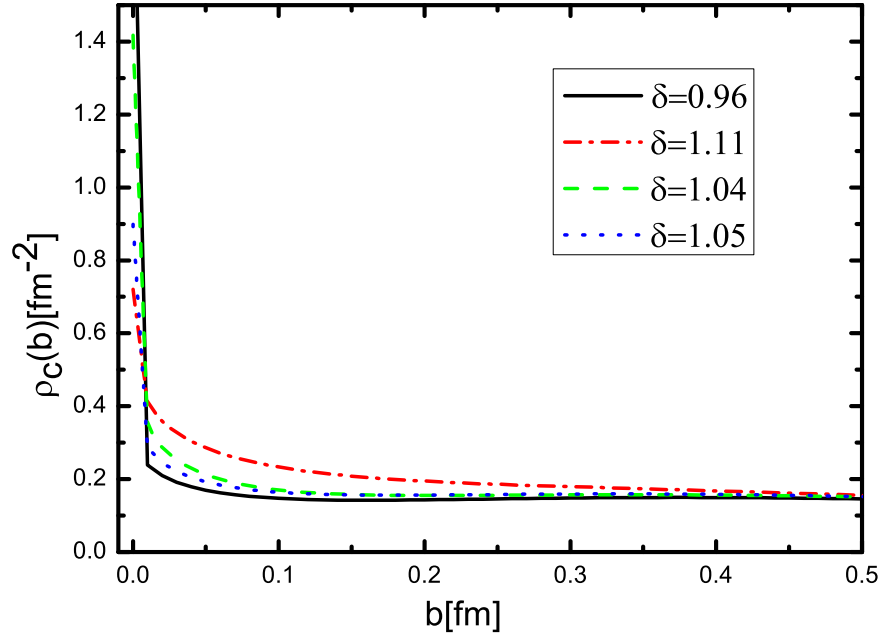


FIG. 2: The obtained transverse charge densities with four different parameterizations.

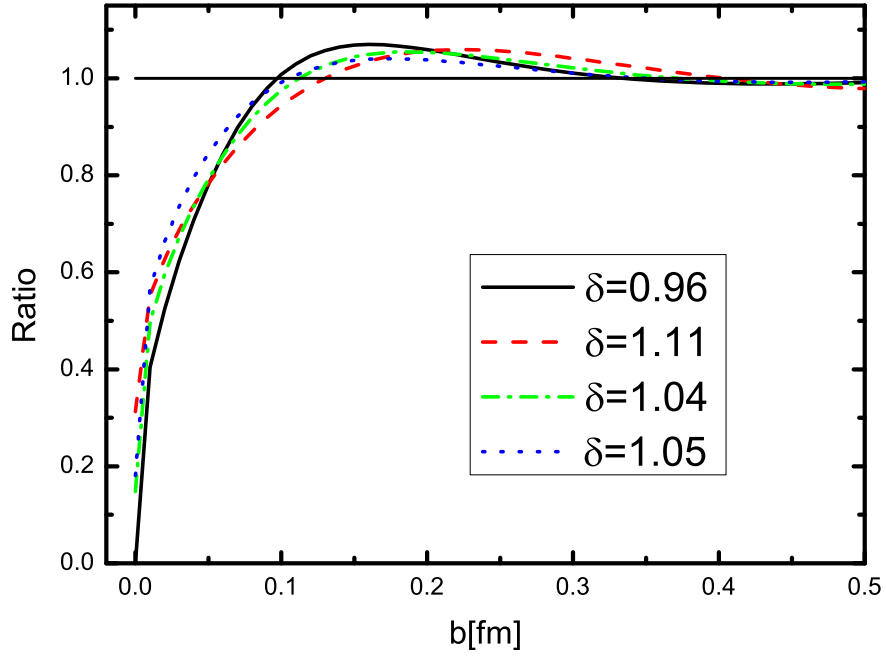


FIG. 3: The ratios of the transverse charge densities contributed by the region of $Q = (0 - 2)$ GeV and by the whole Q region.

TABLE I: The contributions of different Q ranges to $\rho_C(b)$. The unit of $\rho_C(b)$ is fm^{-2} . The deuteron charge form factor is from the parameterizations of Eqs. (4-5): $\alpha = 5.9$, $\beta = -5.2$, $\gamma = 13.9$, $\delta = 0.96$ by Ref. [5]. The notation $a \pm n$ stands for $a \times 10^{\pm n}$.

| Q (GeV) | $b = 0$ fm | $b = 0.01$ fm | $b = 0.02$ fm | $b = 0.1$ fm | $b = 0.5$ fm | $b = 1$ fm |
|---------------|------------------|---------------|---------------|--------------|--------------|------------|
| $0 - \infty$ | <i>divergent</i> | 3.53 - 01 | 2.72 - 01 | 1.44 - 01 | 8.93 - 02 | 8.93 - 02 |
| $0 - 1$ | 1.61 - 01 | 1.61 - 01 | 1.61 - 01 | 1.56 - 01 | 9.15 - 02 | 9.15 - 02 |
| $1 - 2$ | -1.83 - 02 | -1.82 - 02 | -1.82 - 02 | -7.67 - 03 | -2.44 - 03 | -2.44 - 03 |
| $2 - 10$ | 1.16 - 01 | 1.13 - 01 | 1.04 - 01 | -6.15 - 03 | 1.32 - 04 | 1.32 - 04 |
| $10 - \infty$ | <i>divergent</i> | 9.75 - 02 | 2.56 - 02 | 1.38 - 03 | 9.60 - 05 | 9.60 - 05 |

TABLE II: The contributions of different Q ranges to $\rho_C(b)$. The unit of $\rho_C(b)$ is fm^{-2} . The deuteron charge form factor is from the parameterizations of Eqs. (4-5): $\alpha = 5.0$, $\beta = -4.5$, $\gamma = 11.5$, $\delta = 1.11$ by Ref. [5]. The notation $a \pm n$ stands for $a \times 10^{\pm n}$.

| Q (GeV) | $b = 0$ fm | $b = 0.01$ fm | $b = 0.02$ fm | $b = 0.1$ fm | $b = 0.5$ fm | $b = 1$ fm |
|---------------|------------|---------------|---------------|--------------|--------------|------------|
| $0 - \infty$ | 7.20 - 01 | 4.08 - 01 | 3.59 - 01 | 2.33 - 01 | 1.58 - 01 | 8.77 - 02 |
| $0 - 1$ | 2.02 - 01 | 2.02 - 01 | 2.02 - 01 | 2.00 - 01 | 1.59 - 01 | 8.66 - 02 |
| $1 - 2$ | 2.36 - 02 | 2.36 - 02 | 2.35 - 02 | 2.02 - 02 | -7.07 - 03 | 1.25 - 03 |
| $2 - 10$ | 1.24 - 01 | 1.22 - 01 | 1.15 - 01 | 9.45 - 03 | 2.97 - 03 | -2.34 - 04 |
| $10 - \infty$ | 3.71 - 01 | 6.14 - 02 | 1.88 - 02 | 3.76 - 03 | 2.98 - 03 | 6.91 - 05 |

One gets $r_d = 1.966$ fm from the deuteron wave function of Ref. [16]. This value results from the parameterized radial wave function as a discrete superposition of Yukawa-type terms. Here we re-analyze the contributions to the *rms* radius explicitly. The obtained r_d^2 (with D-wave) and r_{ds}^2 (without D-wave) values contributed from the different integral ranges of r are shown in Table V. We see that the integral, in the region of $r = 5 \sim 10$ fm, is also sizeable to r_d^2 . It means the important role of the long tail of the wave function. It is clear that the extrapolation deuteron wave function to the large distance r introduces model dependence and is less known. Moreover, the S-wave function is active at $2 \leq r \leq 20$ fm and its contribution is dominant then. Finally, in the ep and eD elastic scattering, different parameterizations or different model calculations are not the same for the tail of the wave function in the large distance. Our analysis reiterate that the observable r_d^2 contains the sizeable effect from the integral of the larger r space. Although different model calculations can all well reproduce the value of r_d^2 , their extrapolated wave function in the large distance may be different. This phenomenon also happens in the proton case [15].

III. SUMMARY

To summarize, we simply employed the known phenomenological parameterizations of the deuteron charge form factors [5] to get the transverse charge density of the deuteron in the two-dimensional impact space. The obtained densities show quite different behaviors in the limit of $b=0$. This is because the density in the small b region is related to the form factor in the large Q^2 region and all the parameterizations can well reproduce the form factor in the small Q^2 region, but not clear in the large Q^2 region. Therefore, study of the transverse charge density can shed light on

TABLE III: The contributions of different Q ranges to $\rho_C(b)$. The unit of $\rho_C(b)$ is fm^{-2} . The deuteron charge form factor is from the parameterizations of Eqs. (4-5): $\alpha = 5.75$, $\beta = -5.11$, $\gamma = 12.1$, $\delta = 1.04$ by Ref. [5]. The notation $a \pm n$ stands for $a \times 10^{\pm n}$.

| Q (GeV) | $b = 0$ fm | $b = 0.01$ fm | $b = 0.02$ fm | $b = 0.1$ fm | $b = 0.5$ fm | $b = 1$ fm |
|---------------|------------|---------------|---------------|--------------|--------------|------------|
| $0 - \infty$ | 1.42 + 00 | 3.32 - 01 | 2.78 - 01 | 1.70 - 01 | 1.51 - 01 | 9.00 - 02 |
| $0 - 1$ | 1.72 - 01 | 1.71 - 01 | 1.71 - 01 | 1.71 - 01 | 1.48 - 01 | 9.11 - 02 |
| $1 - 2$ | -5.39 - 03 | -5.25 - 03 | -5.23 - 03 | -5.10 - 03 | 1.45 - 03 | -1.31 - 03 |
| $2 - 10$ | 1.09 - 01 | 1.00 - 01 | 9.36 - 02 | -3.54 - 05 | 1.62 - 03 | 1.90 - 05 |
| $10 - \infty$ | 1.14 + 00 | 6.64 - 02 | 1.89 - 02 | 4.28 - 03 | 3.25 - 04 | 7.60 - 05 |

TABLE IV: The contributions of different Q ranges to $\rho_C(b)$. The unit of $\rho_C(b)$ is fm^{-2} . The deuteron charge form factor is from the parameterizations of Eqs. (4-5): $\alpha = 5.50$, $\beta = -4.78$, $\gamma = 12.1$, $\delta = 1.05$ by Ref. [5]. The notation $a \pm n$ stands for $a \times 10^{\pm n}$.

| Q (GeV) | $b = 0$ fm | $b = 0.01$ fm | $b = 0.02$ fm | $b = 0.1$ fm | $b = 0.5$ fm | $b = 1$ fm |
|---------------|------------|---------------|---------------|--------------|--------------|------------|
| $0 - \infty$ | 8.99 - 01 | 2.78 - 01 | 2.45 - 01 | 1.64 - 01 | 1.52 - 01 | 8.96 - 02 |
| $0 - 1$ | 1.73 - 01 | 1.73 - 01 | 1.73 - 01 | 1.72 - 01 | 1.48 - 01 | 9.10 - 02 |
| $1 - 2$ | -1.01 - 02 | -1.01 - 02 | -1.00 - 02 | -9.10 - 3 | 2.94 - 03 | -1.53 - 03 |
| $2 - 10$ | 7.48 - 02 | 7.30 - 02 | 6.80 - 02 | -1.76 - 03 | 9.02 - 04 | 7.80 - 05 |
| $10 - \infty$ | 6.61 - 01 | 5.15 - 02 | 1.46 - 02 | 3.07 - 03 | 2.33 - 04 | 5.40 - 05 |

TABLE V: The different integral ranges of r contribute to r_d^2 and r_{ds}^2 . The notation $a \pm n$ stands for $a \times 10^{\pm n}$.

| r (fm) | $0 - \infty$ | $0 - 1$ | $1 - 2$ | $2 - 5$ | $5 - 10$ | $10 - 20$ | $20 - \infty$ |
|------------------------------|--------------|------------|------------|------------|------------|------------|---------------|
| r_d^2 (fm^2) | 3.855 + 00 | 9.890 - 03 | 1.637 - 01 | 1.341 + 00 | 1.720 + 00 | 6.107 - 01 | 1.994 - 02 |
| r_{ds}^2 (fm^2) | 3.781 + 00 | 9.341 - 03 | 1.509 - 01 | 1.291 + 00 | 1.702 + 00 | 6.084 - 01 | 1.990 - 02 |

the form factor in the large Q^2 region. In conclusion, the transverse charge density in the further measurement is expected to provide the information of the form factors in the large Q^2 region. In addition, we also show that the model calculation of the *rms* radius of the deuteron is also contributed sizeably by the wave function in the large r region. The extrapolation of the wave function in the large distance, therefore, is of great interest.

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